Statistics

Lecture 7

Feb 19-8:47 AM

odds vs. Probability

SG 12

odds in favor of event E are

$$a : b$$

a: # times E happens

b: # times E happens

Always Simplify

I flipped a coin 20 times, it landed tails 8 times.

8 Tails

12 Not Tails

$8 : 12 \rightarrow \boxed{2:3}$

8: Tails; 12: Tails

odds in favor of landing Tails

odds against landing tails

3 : 2

Oct 11-11:33 AM

A box has 4 Red, 6 white, and 20 Blue balls

1) P(Select a red color ball) $= \frac{4}{30} = \boxed{\frac{2}{15}}$

2) odds in favor of Selecting a white ball.

White : # White

6 : 24 → $\boxed{1:4}$

3) odds against Selecting a white ball.

$\boxed{4:1}$

Oct 11-11:38 AM

If odds in favor of event E are $a:b$,

then $P(E) = \frac{a}{a+b}$, $P(\bar{E}) = \frac{b}{a+b}$

Suppose odds in favor of event E are $4:21$

1) odds against event E.

$\boxed{21:4}$

2) $P(E) = \frac{4}{4+21} = \frac{4}{25} = \boxed{.16}$ 3) $P(\bar{E}) = \frac{21}{4+21} = \frac{21}{25} = \boxed{.84}$

$P(E) + P(\bar{E}) = 1$ ✓

Oct 11-11:43 AM

If $P(E)$ is given, then the odds in favor of event E are

$$P(E) : P(\bar{E})$$

Always Simplify

Suppose $P(E) = .025$

1) $P(\bar{E}) = 1 - P(E) = 1 - .025 = .975$

2) odds in favor of event E.

$P(E) : P(\bar{E})$

$.025 : .975$ → $1 : 39$

.025 ÷ .975 Math 1:▶Frac Enter $\frac{1}{39}$

3) odds against event E.

$39 : 1$

Oct 11-11:47 AM

Given $P(A) = .85$

1) $P(\bar{A}) = 1 - .85 = .15$

2) odds in favor of event A

$P(A) : P(\bar{A})$

$.85 : .15$

$17 : 3$

.85 ÷ .15 Math 1:▶Frac Enter $\frac{17}{3}$

3) odds against event A.

$3 : 17$

Oct 11-11:54 AM

Odds & Gambling

True odds

a : b

a → $bet

b → $Net Profit

If odds are 3 : 22

$3 bet

Net Profit $22

Odds in favor of LA Rams to win the Super Bowl this Year are 1 : 499.

$1 bet

$499 Net Profit

Vegas uses different notation (Not true odds)

+150 bet $100, Net $150

−275 bet $275, Net $100

Oct 11-11:59 AM

Multiplication Rule

Keyword AND

Multiplication Event

P(A and B)

A happens first, then B happens

Independent events

outcome of one event does not change the Prob. of next event.

P(Boy) = .5 , P(Girl) = .5

Rolling a fair die

P(get 4) = $\frac{1}{6}$ on every roll

Multiple-choice exam

Each question has 5 choices only one Correct choice Per question, Making guesses

P(guess Correctly) = $\frac{1}{5}$ on every question

Oct 11-12:06 PM

If A and B are independent events,

$P(A \text{ and } B) = P(A) \cdot P(B)$

Ex. $P(A) = .5$, $P(B) = .6$

A & B are independent events.

1) $P(\overline{B}) = 1 - P(B) = \boxed{.4}$

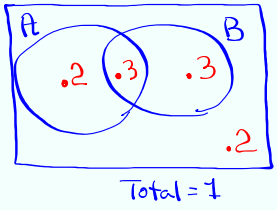
2) $P(A \text{ and } B) = P(A) \cdot P(B) = (.5)(.6) = \boxed{.3}$

3) $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

↑ Addition Rule

$= .5 + .6 - .3 = \boxed{.8}$

4) Draw Venn Diagram



Oct 11-12:13 PM

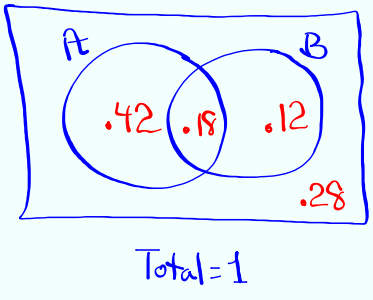
$P(A) = .6$, $P(B) = .3$, A & B are indep. events

1) $P(\overline{B}) = 1 - .3 = \boxed{.7}$

2) $P(A \text{ and } B) = (.6)(.3) = \boxed{.18}$

3) $P(A \text{ or } B) = .6 + .3 - .18 = \boxed{.72}$

4) Draw Venn Diagram



Oct 11-12:20 PM

A loaded coin is tossed twice

P(Tails) = .7

P(Heads) = .3

TT TH HT HH

Sample Space

Complete list of all Possible outcomes

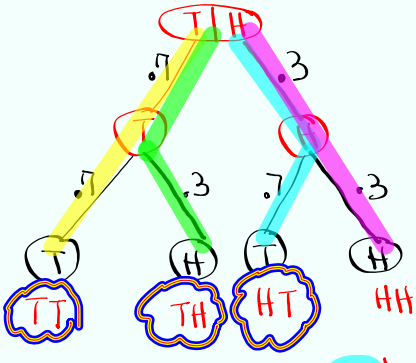
P(TT) = (.7)(.7) = .49 ✓

P(1T & 1H) = 2(.7)(.3) = .42 ✓ Total = 1

P(HH) = (.3)(.3) = .09 ✓

Oct 11-12:25 PM

Multiplication Rule with Tree diagram



P(TT) = (.7)(.7) = .49

P(HT) = (.3)(.7) = .21

P(TH) = (.7)(.3) = .21

P(HH) = (.3)(.3) = .09

P(at least 1 Tails) = 1 − P(HH) =

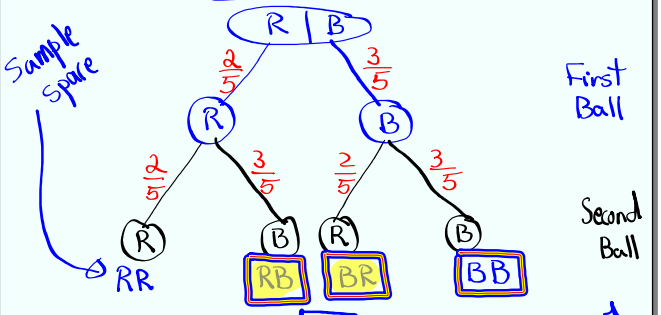
1 ← Total Prob

1 − .09 = .91

Oct 11-12:31 PM

A box has 2 Red and 3 Blue balls.

Take 2 Balls **with replacement**



$P(RR) = \frac{2}{5} \cdot \frac{2}{5} = \frac{4}{25} = .16$

$P(1R \& 1B) = 2 \cdot \frac{2}{5} \cdot \frac{3}{5} = \frac{12}{25} = .48$

$P(BB) = \frac{3}{5} \cdot \frac{3}{5} = \frac{9}{25} = .36$

Total = 1

$P(\text{at least 1 Blue ball}) = 1 - P(\text{No Blue})$

$= 1 - P(RR)$

$= 1 - .16 = .84$

Oct 11-12:38 PM

Dependent Events

SG 13

outcome of one event changes the Prob. of the next event.

when prob. changes → Dependent Events

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$

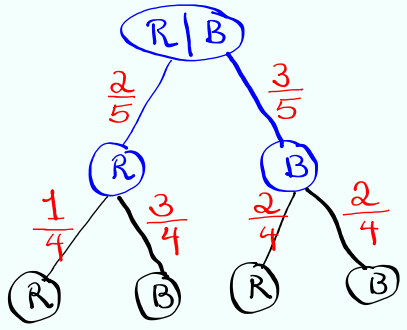
A happens first, then B happens

Given

Oct 11-1:00 PM

A box has 2 red & 3 blue balls.

Select 2 balls without replacement.



$P(RR) = \frac{2}{5} \cdot \frac{1}{4} = \frac{2}{20}$

$P(RB) = \frac{2}{5} \cdot \frac{3}{4} = \frac{6}{20}$

$P(BR) = \frac{3}{5} \cdot \frac{2}{4} = \frac{6}{20}$

$P(BB) = \frac{3}{5} \cdot \frac{2}{4} = \frac{6}{20}$

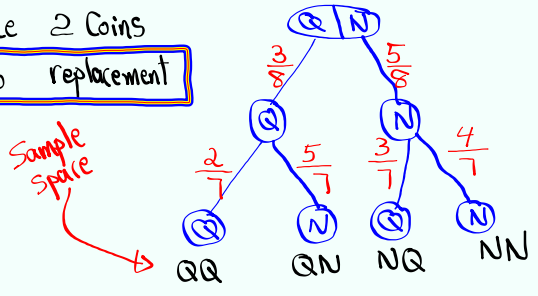
$P(\text{at least 1 Red}) = 1 - P(\text{No Red}) = 1 - P(BB) = 1 - \frac{6}{20} = \frac{14}{20} = \boxed{\frac{7}{10}}$

Oct 11-1:04 PM

A piggy bank has 3 quarters and 5 nickels

Take 2 Coins

No replacement



$P(50¢) = P(QQ) = \frac{3}{8} \cdot \frac{2}{7} = \frac{6}{56}$

$P(30¢) = P(QN \text{ or } NQ) = 2 \cdot \frac{3}{8} \cdot \frac{5}{7} = \frac{30}{56}$

$P(10¢) = P(NN) = \frac{5}{8} \cdot \frac{4}{7} = \frac{20}{56}$

$P(\text{at least 1 nickel}) = 1 - P(\text{No nickels}) = 1 - P(QQ) = 1 - \frac{6}{56} = \frac{50}{56} = \boxed{\frac{25}{28}}$

Oct 11-1:12 PM

A deck of Cards has 40 Cards,
18 red, 10 face, and 3 aces.
Draw 3 Cards, No replacement

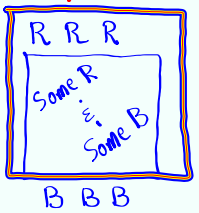
$P(\text{All Red Colors}) = \frac{18}{40} \cdot \frac{17}{39} \cdot \frac{16}{38} = \boxed{\frac{102}{1235}}$

$P(\text{All Black Colors}) = \frac{22}{40} \cdot \frac{21}{39} \cdot \frac{20}{38} = \boxed{\frac{77}{494}}$

$P(\text{All are Same Colors}) = \frac{102}{1235} + \frac{77}{494} = \boxed{\frac{31}{130}}$
RRR OR BBB

$P(\text{are not all Same Color}) = 1 - P(\text{Same}) = 1 - \frac{31}{130} = \boxed{\frac{99}{130}}$

P(at least 1 Red Color Card)



$= 1 - P(\text{No Red})$
$= 1 - P(\text{All Black})$
$= 1 - \frac{77}{494} = \boxed{\frac{417}{494}}$

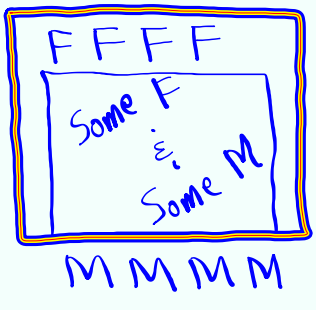
Oct 11-1:21 PM

4 Females, 6 Males
Select 4 people

$P(\text{All Females}) = \frac{4}{10} \cdot \frac{3}{9} \cdot \frac{2}{8} \cdot \frac{1}{7} = \boxed{\frac{1}{210}}$

$P(\text{All Males}) = \frac{6}{10} \cdot \frac{5}{9} \cdot \frac{4}{8} \cdot \frac{3}{7} = \boxed{\frac{1}{14}}$

P(at least 1 Female)



$= 1 - P(\text{No Females})$
$= 1 - P(\text{All Males})$
$= 1 - \frac{1}{14} = \boxed{\frac{13}{14}}$

Oct 11-1:34 PM

Dependent Events

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$

A happens then B happens

Given

Divide by $P(A)$

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

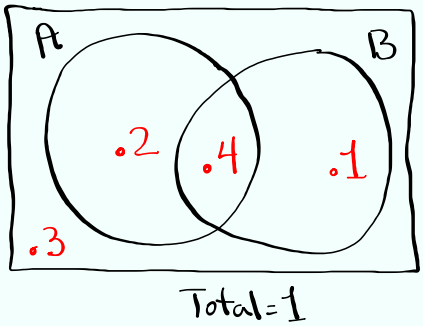
Conditional Prob.

Oct 11-1:52 PM

$P(A) = .6$

$P(B) = .5$

$P(A \text{ and } B) = .4$



$P(A \text{ or } B) = .6 + .5 - .4 = \boxed{.7}$

$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{.4}{.6} = \frac{2}{3} \approx \boxed{.667}$ (AND)

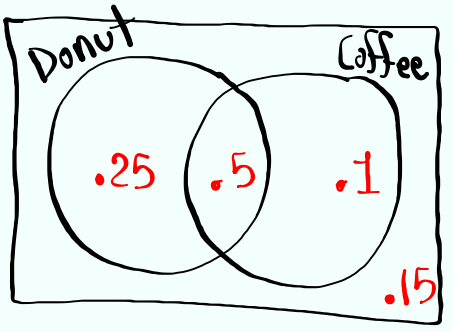
$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{.4}{.5} = \frac{4}{5} = \boxed{.8}$ (AND)

Oct 11-1:55 PM

$P(\text{Donut}) = .75$

$P(\text{Coffee}) = .6$

$P(\text{Donut and Coffee}) = .5$



Total = 1

$$P(\text{Coffee} \mid \text{Donut}) = \frac{P(\text{coffee and Donut})}{P(\text{Donut})} = \frac{.5}{.75} = .667$$

(AND)

$$P(\text{Donut} \mid \text{Coffee}) = \frac{P(C \text{ \& } D)}{P(\text{coffee})} = \frac{.5}{.6} = \frac{5}{6} \approx .833$$

(AND)

Oct 11-2:00 PM

$P(A) = .5$

$P(B) = .6$

$P(B \mid A) = .8$

1) Dependent or independent? ✓

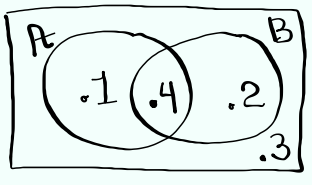
2) $P(A \text{ and } B)$

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$

$$.8 = \frac{P(A \text{ and } B)}{.5}$$

Cross-Multiply

$P(A \text{ and } B) = .4$



(AND)

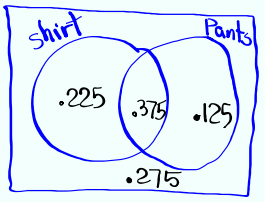
$$P(A \mid B) = \frac{.4}{.6} = \frac{4}{6} = \frac{2}{3} = .667$$

Oct 11-2:06 PM

$P(\text{Shirt}) = .6$

$P(\text{Pants}) = .5$

$P(\text{Shirt} \mid \text{Pants}) = .75$



Total = 1

$P(\text{Shirt and Pants})$

$$P(\text{Shirt} \mid \text{Pants}) = \frac{P(\text{Shirt and Pants})}{P(\text{Pants})}$$

$$.75 = \frac{P(\text{Shirt \& Pants})}{.5}$$

Cross-Multiply

$P(\text{Shirt \& Pants}) = .375$

$$P(\text{Pants} \mid \text{Shirt}) = \frac{.375}{.6} = .625$$

Oct 11-2:12 PM